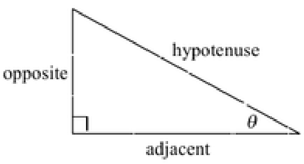
Trig Cheat Sheet

Definition of the Trig Functions

Right triangle definition
For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.

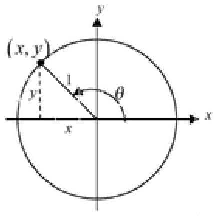


$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\quad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\quad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\quad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

Unit circle definition
For this definition θ is any angle.



$\sin\theta = \frac{y}{1} = y \quad \csc\theta = \frac{1}{y}$

$\cos\theta = \frac{x}{1} = x \quad \sec\theta = \frac{1}{x}$

$\tan\theta = \frac{y}{x} \quad \cot\theta = \frac{x}{y}$

Facts and Properties

Domain
The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, $\quad \theta$ can be any angle
$\cos\theta$, $\quad \theta$ can be any angle

$\tan\theta$, $\quad \theta \neq \left(n + \frac{1}{2}\right)\pi$, $\; n = 0, \pm 1, \pm 2, \ldots$

$\csc\theta$, $\quad \theta \neq n\pi$, $\; n = 0, \pm 1, \pm 2, \ldots$

$\sec\theta$, $\quad \theta \neq \left(n + \frac{1}{2}\right)\pi$, $\; n = 0, \pm 1, \pm 2, \ldots$

$\cot\theta$, $\quad \theta \neq n\pi$, $\; n = 0, \pm 1, \pm 2, \ldots$

Range
The range is all possible values to get out of the function.

$-1 \le \sin\theta \le 1 \qquad \csc\theta \ge 1$ and $\csc\theta \le -1$

$-1 \le \cos\theta \le 1 \qquad \sec\theta \ge 1$ and $\sec\theta \le -1$

$-\infty < \tan\theta < \infty \qquad -\infty < \cot\theta < \infty$

Period
The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$

$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$

$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$

$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$

$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$

$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$

Formulas and Identities

Tangent and Cotangent Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$

$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$

$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Even/Odd Formulas

$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$

Periodic Formulas
If n is an integer.

$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$

$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$

$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$

Double Angle Formulas

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
$\qquad = 2\cos^2\theta - 1$
$\qquad = 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$

Degrees to Radians Formulas
If x is an angle in degrees and t is an angle in radians then

$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180}$ and $x = \frac{180t}{\pi}$

Half Angle Formulas

$\sin^2\theta = \frac{1}{2}\left(1 - \cos(2\theta)\right)$

$\cos^2\theta = \frac{1}{2}\left(1 + \cos(2\theta)\right)$

$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$

Sum and Difference Formulas

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Product to Sum Formulas

$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$

$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$

$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right]$

$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha + \beta) - \sin(\alpha - \beta)\right]$

Sum to Product Formulas

$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$

$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$

$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$

$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$

Cofunction Formulas

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$