

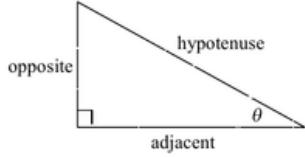
Trig Cheat Sheet

Definition of the Trig Functions

Right triangle definition

For this definition we assume that

$$0 < \theta < \frac{\pi}{2} \text{ or } 0^\circ < \theta < 90^\circ.$$



$$\begin{aligned}\sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} & \csc \theta &= \frac{\text{hypotenuse}}{\text{opposite}} \\ \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} & \sec \theta &= \frac{\text{hypotenuse}}{\text{adjacent}} \\ \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} & \cot \theta &= \frac{\text{adjacent}}{\text{opposite}}\end{aligned}$$

Unit circle definition

For this definition θ is any angle.

A unit circle centered at the origin of a Cartesian coordinate system. A point (x, y) is located on the circle in the first quadrant. The angle θ is measured counter-clockwise from the positive x-axis to the line segment connecting the origin to the point (x, y) . The radius r is labeled as 1. The coordinates x and y are shown, along with the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$.

$$\begin{aligned}\sin \theta &= \frac{y}{r} = y & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} = x & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$$\begin{aligned}\sin \theta, \quad \theta &\text{ can be any angle} \\ \cos \theta, \quad \theta &\text{ can be any angle} \\ \tan \theta, \quad \theta &\neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \csc \theta, \quad \theta &\neq n\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \sec \theta, \quad \theta &\neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \cot \theta, \quad \theta &\neq n\pi, \quad n = 0, \pm 1, \pm 2, \dots\end{aligned}$$

Range

The range is all possible values to get out of the function.

$$\begin{aligned}-1 \leq \sin \theta \leq 1 & \quad \csc \theta \geq 1 \text{ and } \csc \theta \leq -1 \\ -1 \leq \cos \theta \leq 1 & \quad \sec \theta \geq 1 \text{ and } \sec \theta \leq -1 \\ -\infty < \tan \theta < \infty & \quad -\infty < \cot \theta < \infty\end{aligned}$$

Period

The period of a function is the number, T , such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\begin{aligned}\sin(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \cos(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \tan(\omega\theta) &\rightarrow T = \frac{\pi}{\omega} \\ \csc(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \sec(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \cot(\omega\theta) &\rightarrow T = \frac{\pi}{\omega}\end{aligned}$$

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Formulas and Identities

Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Reciprocal Identities

$$\begin{aligned}\csc \theta &= \frac{1}{\sin \theta} & \sin \theta &= \frac{1}{\csc \theta} \\ \sec \theta &= \frac{1}{\cos \theta} & \cos \theta &= \frac{1}{\sec \theta} \\ \cot \theta &= \frac{1}{\tan \theta} & \tan \theta &= \frac{1}{\cot \theta}\end{aligned}$$

Pythagorean Identities

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \tan^2 \theta + 1 &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \csc^2 \theta\end{aligned}$$

Even/Odd Formulas

$$\begin{aligned}\sin(-\theta) &= -\sin \theta & \csc(-\theta) &= -\csc \theta \\ \cos(-\theta) &= \cos \theta & \sec(-\theta) &= \sec \theta \\ \tan(-\theta) &= -\tan \theta & \cot(-\theta) &= -\cot \theta\end{aligned}$$

Periodic Formulas

$$\begin{aligned}\text{If } n \text{ is an integer.} \\ \sin(\theta + 2\pi n) &= \sin \theta & \csc(\theta + 2\pi n) &= \csc \theta \\ \cos(\theta + 2\pi n) &= \cos \theta & \sec(\theta + 2\pi n) &= \sec \theta \\ \tan(\theta + \pi n) &= \tan \theta & \cot(\theta + \pi n) &= \cot \theta\end{aligned}$$

Double Angle Formulas

$$\begin{aligned}\sin(2\theta) &= 2 \sin \theta \cos \theta \\ \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \\ &= 1 - 2 \sin^2 \theta \\ \tan(2\theta) &= \frac{2 \tan \theta}{1 - \tan^2 \theta}\end{aligned}$$

Degrees to Radians Formulas

$$\begin{aligned}\text{If } x \text{ is an angle in degrees and } t \text{ is an angle in radians then} \\ \frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}\end{aligned}$$

Half Angle Formulas

$$\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Sum and Difference Formulas

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\ \tan(\alpha \pm \beta) &= \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}\end{aligned}$$

Product to Sum Formulas

$$\begin{aligned}\sin \alpha \sin \beta &= \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)] \\ \cos \alpha \cos \beta &= \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)] \\ \sin \alpha \cos \beta &= \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)] \\ \cos \alpha \sin \beta &= \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]\end{aligned}$$

Sum to Product Formulas

$$\begin{aligned}\sin \alpha + \sin \beta &= 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \\ \sin \alpha - \sin \beta &= 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right) \\ \cos \alpha + \cos \beta &= 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) \\ \cos \alpha - \cos \beta &= -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)\end{aligned}$$

Cofunction Formulas

$$\begin{aligned}\sin\left(\frac{\pi}{2} - \theta\right) &= \cos \theta & \cos\left(\frac{\pi}{2} - \theta\right) &= \sin \theta \\ \csc\left(\frac{\pi}{2} - \theta\right) &= \sec \theta & \sec\left(\frac{\pi}{2} - \theta\right) &= \csc \theta \\ \tan\left(\frac{\pi}{2} - \theta\right) &= \cot \theta & \cot\left(\frac{\pi}{2} - \theta\right) &= \tan \theta\end{aligned}$$

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