

TRIGONOMETRIC IDENTITIES

Co-function Identities

$$\begin{aligned}\sin \theta &= \cos (\pi / 2-\theta) \\ \sec \theta &= \csc (\pi / 2-\theta) \\ \tan \theta &= \cot (\pi / 2-\theta)\end{aligned}$$

Negative Angle Identities

$$\begin{aligned}\sin (-\theta) &= -\sin \theta & \csc (-\theta) &= -\csc \theta \\ \cos (-\theta) &= \cos \theta & \sec (-\theta) &= \sec \theta \\ \tan (-\theta) &= -\tan \theta & \cot (-\theta) &= -\cot \theta\end{aligned}$$

Addition and Subtraction Identities

$$\sin (A+B)=\sin A \cos B+\cos A \sin B$$

$$\cos (A+B)=\cos A \cos B-\sin A \sin B$$

$$\tan (A+B)=\frac{\tan A+\tan B}{1-\tan A \tan B}$$

$$\sin (A-B)=\sin A \cos B-\cos A \sin B$$

$$\cos (A-B)=\cos A \cos B+\sin A \sin B$$

$$\tan (A-B)=\frac{\tan A-\tan B}{1+\tan A \tan B}$$

Double-Angle Identities

$$\sin 2 \theta=2 \sin \theta \cos \theta$$

$$\cos 2 \theta=\cos ^2 \theta-\sin ^2 \theta$$

$$=2 \cos ^2 \theta-1$$

$$=1-2 \sin ^2 \theta$$

$$\tan 2 \theta=\frac{2 \tan \theta}{1-\tan ^2 \theta}$$

Product Identities

$$\sin A \cos B=\frac{1}{2}(\sin (A+B)+\sin (A-B))$$

$$\cos A \sin B=\frac{1}{2}(\sin (A+B)-\sin (A-B))$$

$$\cos A \cos B=\frac{1}{2}(\cos (A+B)+\cos (A-B))$$

$$\sin A \sin B=\frac{1}{2}(\cos (A-B)-\cos (A+B))$$

Supplement Angle Identities

$$\begin{aligned}\sin (\pi-\theta) &= \sin \theta & \csc (\pi-\theta) &= \csc \theta \\ \cos (\pi-\theta) &= -\cos \theta & \sec (\pi-\theta) &= -\sec \theta \\ \tan (\pi-\theta) &= -\tan \theta & \cot (\pi-\theta) &= -\cot \theta\end{aligned}$$

$$\begin{aligned}\sin (\pi+\theta) &= -\sin \theta & \csc (\pi+\theta) &= -\csc \theta \\ \cos (\pi+\theta) &= -\cos \theta & \sec (\pi+\theta) &= -\sec \theta \\ \tan (\pi+\theta) &= \tan \theta & \cot (\pi+\theta) &= \cot \theta\end{aligned}$$

Quotient Identities

$$\begin{aligned}\tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta} \\ \sec \theta &= \frac{1}{\cos \theta} & \csc \theta &= \frac{1}{\sin \theta}\end{aligned}$$

Pythagorean Identities

$$\sin ^2 \theta+\cos ^2 \theta=1$$

$$\tan ^2 \theta+1=\sec ^2 \theta$$

$$\cot ^2 \theta+1=\csc ^2 \theta$$

Half-Angle Identities

$$\sin \frac{\theta}{2}=\pm \sqrt{\frac{1-\cos \theta}{2}}$$

$$\cos \frac{\theta}{2}=\pm \sqrt{\frac{1+\cos \theta}{2}}$$

$$\tan \frac{\theta}{2}=\pm \sqrt{\frac{1-\cos \theta}{1+\cos \theta}}$$

Sum Identities

$$\sin A+\sin B=2 \sin \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right)$$

$$\sin A-\sin B=2 \cos \left(\frac{A+B}{2}\right) \sin \left(\frac{A-B}{2}\right)$$

$$\cos A+\cos B=2 \cos \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right)$$

$$\cos A-\cos B=-2 \sin \left(\frac{A+B}{2}\right) \sin \left(\frac{A-B}{2}\right)$$