

| Use this substitution | in this expression | ... here's the arithmetic  | simplified radical | domain of $t$                                      |
|-----------------------|--------------------|--|--------------------|--|
| $x = a \sin(t)$       | $\sqrt{a^2 - x^2}$ | $\sqrt{a^2 - a^2 \sin^2 t} = \sqrt{a^2(1 - \sin^2 t)} = \sqrt{a^2 \cos^2 t}$ | $= a \cos(t)$      | $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ |
| $x = a \tan(t)$       | $\sqrt{a^2 + x^2}$ | $\sqrt{a^2 + a^2 \tan^2 t} = \sqrt{a^2(1 + \tan^2 t)} = \sqrt{a^2 \sec^2 t}$ | $= a \sec(t)$      | $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ |
| $x = a \sec(t)$       | $\sqrt{x^2 - a^2}$ | $\sqrt{a^2 \sec^2 t - a^2} = \sqrt{a^2(\sec^2 t - 1)} = \sqrt{a^2 \tan^2 t}$ | $= a \tan(t)$      | $t \in [0, 2\pi]$                                  |