ABSTRACT ALGEBRA FORMULAS

RECIPROCAL IDENTITIES

Group Definition: $(G, *)$
Closure: $a * b \in G$
Associativity: $(a * b) * c = a * (b * c)$
Identity Element: $a * e = e * a = a$
Inverse Element: $a * a^{-1} = e$
Abelian Group: $a * b = b * a$

GRUP DEFINITION

Order of Group: $\frac{a}{a \cdot b}$
Cyclic Element: $\sqrt{2} = \frac{\overline{e^2}}{a}$
Cyclic Element: $\frac{a}{a \cdot b} = \frac{a}{b}$

NORMAL SUBGROUP

Normal Subgroup: $aH = Ha$
Quotient Group: G/H
Homomorphism: $f(ab) = f(a)f(b)$
Kernel: $\ker(f) = \{a : f(a) = e\}$
Image: $\mathrm{Im}(f) = f(G)$
Isomorphism: Bijective homomorphism

FIELD

Field: Every nonzero has inverse
Ideal: Subset closed under operations
Principal Ideal: (a)
PID: Every ideal principal
UFD: Unique factorization
Polynomial Ring: $R[x]$

ORDER OF GROUP

Order of Group: $\frac{G \times \langle G \rangle}{a^n = e}$
Order of Element: $a^n = e$
Subgroup Test: Closed + inverses
Cyclic Group: $G = \langle a \rangle$
Lagrange's Theorem: $\frac{HR \to G}{HT - HI}$
Cosets: aH or Ha

FIRST ISOMORPHISM

First Isomorphism: $G/\ker(f) \cong \mathrm{Im}(f)$
Ring Definition: $(R, +, \times)$
Commutative Ring: $ab = ba$
Unity Element: $1 \in R$
Zero Divisor: $ab = 0,\ a \neq 0,\ b \neq 0$
Integral Domain: No zero divisors

DEGREE OF POLYNOMIAL

Degree of Polynomial: Highest power
Root Condition: $f(a) = 0$
Field Extension: E/F
Degree of Extension: $[E:F]$
Galois Group: $\mathrm{Aut}(E/F)$
Cayley Table: Operation table

DIRECT PRODUCT

Direct Product: $G \times H$
Ring Homomorphism: $f(a+b) = f(a) + f(b)$
Unit Element: a^{-1} exists