COMPLEX ANALYSIS FORMULAS

COMPLEX NUMBER

Complex Number: $z = x + iy$

Modulus: $|z| = \sqrt{x^2 + y^2}$

Argument: $\arg(z) = \tan^{-1}\left(\frac{y}{x}\right)$

Conjugate: $\overline{z} = x - iy$

Polar Form: $z = r(\cos\theta + i\sin\theta)$

Euler's Formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

EXPONENTIAL FORM

Exponential Form: $z = re^{i\theta}$

De Moivre's Theorem:

$$z^n = r^n e^{in\theta}$$

nth Roots: $z^{\frac{1}{1/n}} = r^{\frac{1}{1/n}} e^{\frac{i(\theta+2k\pi)}{n}}$

Triangle Inequality:

$$|z_1 + z_2| \leq |z_1| + |z_2|$$

Analytic Function:

$f'(z)$ exists

Holomorphic: Complex differentiable everywhere

CAUCHY-RIEMANN EQNS

Cauchy-Riemann Eqns:

$$u_x = v_y,\ u_y = -v_x$$

Harmonic Function:

$$\nabla^2 u = 0$$

Entire Function:

Analytic everywhere

Singularities

Points where f undefined

Removable Singularity:

Limit exists

Pole: $f(z) \to \infty$

ESSENTIAL SINGULARITY

Essential Singularity

Irregular behavior

Laurent Series

$$f(z) = \sum_{n=-\infty}^{\infty} a_n (z - z_0)^n$$

Taylor Series

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$$

Contour Integral: $\oint f(z)dz$

Cauchy Integral Theorem

$$\oint f(z)dz = 0$$

Cauchy Integral Formula

$$f(a) = \frac{1}{2\pi i} \oint \frac{f(z)}{z-a} dz$$

HIGHER DERIVATIVES

Higher Derivatives:

$$f^{(n)}(a) = \frac{n!}{2\pi i} \oint \frac{f(z)}{(z-a)^{n+1}} dz$$

Residue: Coefficient of $\frac{1}{z-a}$

Residue Theorem:

$$\oint f(z)dz = 2\pi i \sum \text{Res}$$

Liouville's Theorem:

Bounded entire $\Rightarrow$ constant

Maximum Modulus:

Max on boundary

Argument Principle:

$$Z - P = \frac{1}{2\pi i} \oint \frac{f'(z)}{f(z)} dz$$

ROUCHE'S THEOREM

Rouche's Theorem

Compare functions

Conformal Map

Angle preserving

Mobius Transform

$$f(z) = \frac{az+b}{cz+d}$$

Open Mapping

Open sets remain open