DIFFERENTIAL EQUATIONS FORMULAS

BASIC DIFFERENTIAL EQUATION

Basic Differential Equation: $\frac{dy}{dx} = f(x,y)$

Order of DE: Highest derivative

First Order Linear DE: $\frac{dy}{dx} + P(x)y = Q(x)$

Integrating Factor (IF): $IF = e^{\int P(x)dx}$

Solution (Linear DE): $y \cdot IF = \int Q(x) \cdot IF\, dx$

Separable Equation: $\frac{dy}{dx} = g(x)h(y)$

SOLUTION (SEPARABLE)

Solution (Separable): $\int \frac{dy}{h(y)} = \int g(x)dx$

Homogeneous DE: $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$

Exact Equation: $Mdx + Ndy = 0$

Solution (Exact):

$$\int Mdx + \int \left(N - \frac{\partial}{\partial y}\int Mdx\right)dy$$

Bernoulli Equation: $\frac{dy}{dx} + Py = Qy^n$

Substitution: $v = y^{1-n}$

SECOND ORDER DE

Second Order DE: $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$

Characteristic Equation: $m^2 + am + b = 0$

Distinct Roots: $y = C_1e^{m_1x} + C_2e^{m_2x}$

Repeated Roots: $y = (C_1 + C_2x)e^{mx}$

Complex Roots:

$$y = e^{\alpha x}(C_1\cos\beta x + C_2\sin\beta x)$$

Non-Homogeneous DE: $LHS = RHS$

PARTICULAR SOLUTION

Particular Solution: $y = y_h + y_p$

Method of Undetermined Coefficients: Guess y_p form

Variation of Parameters: $y_p = u_1y_1 + u_2y_2$

Laplace Transform: $\mathbb{L}\{f(t)\} = \int_0^\infty e^{-st}f(t)dt$

$\mathbb{L}\{1\}$: $\frac{1}{s}$

$\mathbb{L}\{t^n\}$: $\frac{n!}{s^{n+1}}$

L{E^AT}

$\mathbb{L}\{e^{at}\}$: $\frac{1}{s-a}$

$\mathbb{L}\{\sin(at)\}$: $\frac{a}{s^2+a^2}$

$\mathbb{L}\{\cos(at)\}$: $\frac{s}{s^2+a^2}$

Inverse Laplace: $\mathbb{L}^{-1}\{F(s)\}$

Convolution:

$$(f*g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau$$

Euler Method: $y_{n+1} = y_n + hf(x_n, y_n)$

IMPROVED EULER

Improved Euler:

$$y_{n+1} = y_n + \frac{h}{2}[f(x_n,y_n) + f(x_{n+1},y_{n+1})]$$

Runge-Kutta (RK4):

$$y_{n+1} = y_n + y_n + \frac{k_1+2k_2+2k_3+k_4}{6}$$

Logistic Growth: $\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$

Solution Logistic: $P = \frac{K}{1 + Ae^{-rt}}$

Exponential Growth: $\frac{dP}{dt} = kP$

Solution: $P = P_0e^{kt}$