LINEAR ALGEBRA FORMULAS

VECTOR FORM

Vector Form: $\mathbf{v} = (v_1, v_2, v_3)$

Vector Magnitude: $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

Unit Vector: $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$

Dot Product: $\mathbf{a} \cdot \mathbf{b} = x_1 x_2 + y_1 y_2 + z_1 z_2$

Angle Between Vectors: $\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \cdot \|\mathbf{b}\|}$

Cross Product: $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$

CROSS PRODUCT MAGNITUDE

Cross Product Magnitude:
$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \cdot \|\mathbf{b}\| \sin\theta$

Vector Projection: $\text{proj}_{a}\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|^2 a}\mathbf{a}$

Matrix Addition: $A + B = [a_{ij} + b_{ij}]$

Scalar Multiplication: $kA = [k \cdot \mathbf{a}_{ij}]$

Matrix Multiplication: $AB = [c_{ij}]$

where $c_{ij} = \sum_{k} a_{ik} b_{kj}$

Identity Matrix: $AI = IA = A$

TRANSPOSE

Transpose: $(A^T)_{ij} = a_{ji}$

Determinant (2×2): $\det(A) = ad - bc$

Determinant (3×3): $\det(A) =$
$\det(A) = a(ei - fh) - b(di - fg) + c(dh - eg)$

Inverse Matrix: $A^{-1} = \frac{\text{adj}(A)}{\det(A)}$

Condition for Inverse: $\det(A) \neq 0$

System of Equations: $AX = B$

CRAMER'S RULE

Cramer's Rule: $x_i = D_i / D$

Rank of Matrix: $\frac{\text{Number of independent}}{\text{rows/columns}}$

Eigenvalue Equation: $A\mathbf{v} = \lambda\mathbf{v}$

Characteristic Equation: $\det(A - \lambda I) = 0$

Trace of Matrix: $\text{tr}(A) = \sum_{i=1}^{n} a_{ii}$

Orthogonality: $\mathbf{a} \cdot \mathbf{b} = 0$

SYMMETRIC MATRIX

Symmetric Matrix: $A = A^T$

Skew-Symmetric: $A^T = -A$

Orthogonal Matrix: $A^T A = I$

Diagonalization: $A = PDP^{-1}$

Rank-Nullity:
$\text{Rank}(A) + \text{Nullity}(A) = n$ (columns)

Norm (L2): $\|\mathbf{x}\|_2 = \sqrt{\sum_{i=1}^{n} x_i^2}$

NORM (L1)

Norm (L1): $\|\mathbf{x}\|_1 = \sum_{i=1}^{n} |x_i|$

Norm (L∞): $\|\mathbf{x}\|_\infty = \max_i |x_i|$

Inner Product: $\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^T \mathbf{b}$

Cauchy-Schwarz: $|\langle \mathbf{a}, \mathbf{b} \rangle| \leq \|\mathbf{a}\| \cdot \|\mathbf{b}\|$

Gram-Schmidt:
Orthogonalization process

Least Squares: $(A^T A)\mathbf{x} = A^T \mathbf{b}$

SVD

SVD: $A = U\Sigma V^T$