REAL ANALYSIS FORMULAS

LIMIT DEFINITION

Limit Definition: $\lim_{x \to a} f(x) = L$

Epsilon-Delta: $\forall \varepsilon > 0 \, \exists \delta > 0$

Sequence Convergence: $\lim_{n \to \infty} a_n = L$

Uniqueness of Limit: Limit is unique

Bounded Sequence:

Monotone Sequence:
Increasing or decreasing

MEAN VALUE THEOREM

Mean Value Theorem: $f'(c) = \frac{f(b) - f(a)}{b - a}$

Rolle's Theorem: $f(a) = f(b) \Rightarrow f'(c) = 0$

Taylor's Theorem: $f(x) = \sum + R_n$

Riemann Integral: $\int f(x)dx$

Darboux Sums: Upper & lower sums

Integrability: Upper = lower sum

MONOTONE CONVERGENCE

Monotone Convergence:
Bounded + monotone ⇒ convergent

Cauchy Sequence:

Completeness:
Every Cauchy converges

Bolzano-Weierstrass:
Bounded ⇒ convergent subsequence

Limit Superior: $\limsup a_n$

Limit Inferior: $\liminf a_n$

FUNDAMENTAL THEOREM

Fundamen Theorem: $\int_a^b f(x)dx = F(b) - F(a)$

Improper Integral: Limit of integral

Comparison Test:
Compare with known

Ratio Test: $\lim$

Root Test: $\lim \sqrt{na_n}$

Alternating Test: Decreasing + $\to 0$

CONTINUITY

Continuity: $\lim_{x \to a} f(x) = f(a)$

Uniform Continuity:
Same δ works $\forall x$

Intermediate Value:
$f(a) < k < f(b) \Rightarrow \exists c$

Extreme Value: Max & Min exist

Derivative Definition:
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Differentiability: ⇒ continuity

ABSOLUTE CONVERGENCE

Absolute Convergence:

Conditional Convergence:
Converges but not absolute

Uniform Convergence:
Same convergence speed

Pointwise Convergence:
Each point separately

Weierstrass M-Test:

Lipschitz Condition:

SUPREMUM

Supremum: Least upper bound

Infimum: Greatest lower bound