TOPOLOGY FORMULAS

TOPOLOGICAL SPACE

Topological Space: (X, τ)

Open Set: $U \in \tau$

Closed Set: $X - U$

Closure: $\overline{\text{cl}}(A)$

Interior: $\text{int}(A)$

Boundary: $\partial A = \overline{\text{cl}}(A) - \text{int}(A)$

QUOTIENT TOPOLOGY

Quotient Topology: $\tau = \{U \subseteq Y : f^{-1}(U) \text{ open}\}$

Continuous Function: $f^{-1}(U)$ open

Homeomorphism: Bijective + continuous + inverse continuous

Topological Property: Invariant under homeomorphism

Open Map: $f(U)$ open

Closed Map: $f(F)$ closed

HEINE-BOREL

Heine-Borel: Closed & bounded $\Rightarrow$ compact

Sequential Compactness: Every sequence has convergent subsequence

Connected Space: Cannot split into disjoint open sets

Path Connected: Any two points connected by path

Components: Maximal connected

Hausdorff Space: Points separable by neighborhoods

OPEN BALL

Open Ball: $B(x, r) = \{y : d(x, y) < r\}$

Complete Space: Every Cauchy sequence converges

Baire Category: Not union of nowhere dense sets

Fixed Point (Banach): Contraction Contraction $\Rightarrow$ unique point

Fundamental Group: $\pi_1(X)$

Simply Connected: No holes

LIMIT POINT

Limit Point: Every neighborhood intersects A

Dense Set: $\text{cl}(A) = X$

Nowhere Dense: $\text{int}(\text{cl}(A)) = \emptyset$

Basis: Collection generating topology

Subspace Topology: $\tau_Y = \{U \cap Y\}$

Product Topology: $\tau = \prod \tau_i$

EMBEDDING

Embedding: Injective homeomorphism onto image

First Countable: Countable neighborhood basis

Second Countable: Countable base

Separable Space: Countable dense subset

Lindelöf Space: Every cover has countable subcover

Compact Space: Every open cover has finite subcover

REGULAR SPACE

Regular Space: Point & closed set separable

Normal Space: Two closed sets separable

Urysohn Lemma: Separate sets via function

Tietze Extension: Extend continuous functions

Tychonoff Theorem: Product of compact spaces compact

Metric Space: (X, d)

COVERING SPACE

Covering Space: Local homeomorphism
Local homeomorphism

Homotopy: Continuous deformation

Homology: Algebraic invariants

Euler Characteristic: $V - E + F$